Impacts of the Atlatl & Proving Their Effectiveness Mathematically

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for the 2021-2022 World View Global Fellows Program

Overview of Lesson: The purpose of this lesson is to introduce students to the atlatl, an instrument or spear-thrower used by American Indians in North Carolina to aid in hunting, and to examine this instrument from a mathematical perspective. In doing so, students will use prior knowledge of circles to understand the concept of arc length as well as construct the calculation necessary to determine arc length. This will lesson also lead to an understanding of radian measures, what they represent, and how these are connected to degree measurements.

About the Project: American Indians lived in the area we now call North Carolina for at least 15,000 years. Archaeologists study the remnants of their communities to learn who these people were and how they lived and prospered for thousands of years. The lesson presented below was developed as part of the 2021-2022 UNC World View Fellows Program, *Exploring Indigenous Cultures: Ancient North Carolinians, Past and Present*. It is one in a series of lessons that the Fellows created for K-12 schools and community colleges to help students learn about the ancient peoples that lived here and those who represent today's vibrant American Indian populations. Lessons connect past to present day by exploring multiple resources within the Ancient North Carolinians: A Virtual Museum of North Carolina Archaeology website to examine how communities changed over time and what influenced these changes. Understanding past Indigenous lifeways—their complexity, resiliency, and vitality—allows for a greater appreciation of the contributions American Indians made to the past and continue to make to the present and future of North Carolina.

Suggested Grade Levels: 10^{th} & 11^{th} (standard, inclusion, and honors)

Subject: NC Math 3

Corresponding National and State of North Carolina Standards:
NC Math 3 Standards:
NC.M3.F-TF.1: Understand radian measure of an angle as:
- The ratio of the length of an arc on a circle is subtended by the angle to its radius
- A dimensionless measure of length defined by the quotient of arc length and radius that is a real number
- The domain for trigonometric functions

NC.M3.G-C.5: Using similarity, demonstrate that the length of an arc, *s*, for a given central angle is proportional to the radius, *r*, of a circle. Define radian measure of the central angle as the ratio of the length of the arc to the radius of the circle, *s/r*. Find arc lengths and areas of sectors of circles.

Essential Questions:
1. What is an *atlatl*? How did it impact Indigenous cultures in North Carolina?
2. How does changing the length of a radius impact a projectile?
3. How does radius length allow you to define arc length?

Lesson Objectives:

1. Students will recognize an atlatl, understand its place and significance in the development of Indigenous cultures in North Carolina, and how its benefits can be modeled using mathematics.
2. Students will understand what a radian is, the connection between the radius and arc length, and how to calculate arc length using a central angle and a radius.

Background information for the educator:
The atlatl was an instrument or spear-thrower used by American Indians in North Carolina during the Archaic period to aid in hunting. The atlatl allowed for an extension of the throwing arm (radius), which increased speed and distance when the projectile (stone-tipped spear) was moved through an equivalent central angle (but larger arc). View image here: https://64parishes.org/entry-image/hunting-with-an-atlatl.

Discussion Questions:

- Questions to use during instruction: Consider the atlatl's purpose → ask students:
    - Why do you think this was created?
    - What benefits do you foresee?
- Follow-up question: What allowed this tool to be beneficial?
- Guiding questions and simulations using the *ChuckIt*! ball launcher (see materials list at end) can be utilized to help spark student ideas if those are not readily flowing - these can include:
    - Explore image available here and utilize photographic literacy to look for context clues: What do you see going on in this photo?
    - Simulation with an atlatl-like implement and without: How do we see the atlatl impacting the projectile (ie. the spear)? What kind of impact would this have?

Student Activities:

Day 1

- (~15 minutes) Review previous day's assignment, address any follow up questions, and allow for any clarification or extension.
- (~15 minutes) Utilize the image above to start the discussion questions:
    - Why do you think the atlatl was created?
    - What benefits do you foresee? (Students can be prompted to use the image for context clues)
    - After initial student responses, students can be prompted with the following questions pertaining to the benefits students predicted:
        - Why do you think the atlatl was beneficial?
            - The class can step outside to demonstrate throwing a tennis ball by hand and throwing with the atlatl-like instrument (*ChuckIt*! ball launcher) to demonstrate effectiveness
            - Can have more than one student take part, first throwing the tennis ball with just their arm, then throwing with the *ChuckIt*! ball launcher to

demonstrate the additional distance achieved

- (~20 minutes) Students will pair up and read The Pathfinders from the Ancient North Carolinians website. Instruct students to look for supporting evidence or additional explanation to the answers they have created to the earlier questions and take notes on this information. From the website:
    - Atlatl is a wooden stick with a handle on one end and hood on the other
    - Acts like a lever (can connect back to simple machines in science)
    - Greatly increased accuracy and force of spear
    - *Crucial note*: The atlatl extends the throwing arm of the person trying to throw the spear
        - Some diagrams and sketches are given in the reading, but should also refer back to the Ancient North Carolinians website for photos of atlatl stones and spear points, as well as a YouTube video explaining the use of atlatl weights:
            - Complete bannerstone* found in Stanley Co
            - Bannerstone fragments found in Stanley Co
            - Spear points found at Garden Creek Mound #2 (a)
            - Spear points found at Garden Creek Mound #2 (b)
            - YouTube Video (go to about 3:50 for bannerstone discussion

              **Bannerstones* are sometimes called 'atlatl weights', however their actual use is still debated. For more information, read "Why did prehistoric Native Americans fashion the enigmatic objects known as bannerstones?"
        - Worth noting that at Town Creek Indian Mound in Mt. Gilead (Montgomery Co), NC, you can practice throwing an atlatl
  - From these notes, particularly the last point, the mathematics of the atlatl will be explored
- (~30 minutes) Students will be introduced to vocabulary:
  - Radius – distance from center of a circle to the edge
    - Every radius in a circle has the same length
  - Diameter – distance from the edge of the circle, through the center, to the opposite side
    - 2 radii = 1 diameter
    - Every diameter in a circle has the same length
  - Circumference – the distance around the outside of a circle (similar to *perimeter* of polygons)
    - C = $2\pi r$ or πd
  - Arc – a snippet or section of the circumference (not the entire circumference)
  - Arc length – the measurement of the arc
  - Central angle – the angle formed with a vertex at the center of the circle
    - Recall from earlier lessons that the angle measure of the central angle is equal to the angle measure of the intercepted arc

- Radian – the measurement of an angle that using the proportion $\frac{arc\ length}{radius}$
  - Example: A circle with a radius of 1 meter and a circumference of 2π m – this means that a full rotation around circle can be measured with an angle that is 360 degrees or 2π radians
    - Therefore, 2π = 360 degrees
    - What does this mean for 180 degrees (half the circle)?
      - Answer: equivalent to $\frac{2\pi}{2}$ or π radians
      - Therefore, 180 degrees = π radians
      - This conversion factor can be used to convert any degree measure to radians: degrees * $\frac{\pi}{180}$ = radians

- Desmos Vocabulary Matching Activity – take final ~10 minutes of class to have students complete this activity to ensure proper understanding of vocabulary, concepts, and equations needed going forward

Day 2

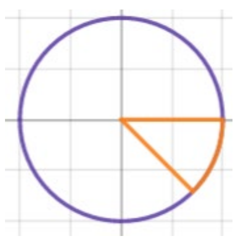
- (~10 minutes) Review answers to the Desmos activity, discuss any misconceptions, and ensure all terms are well understood going into today's discussion
- (~30 minutes) How will we calculate the length of an arc?
  - Arc is part of circumference – what is another, more mathematical term, for 'part'? Answer: fraction – we'll need to find a fraction of the circumference
    - How will we set up this fraction?
      Answer: To be a fraction of the circumference, we need to use the entire circumference as the denominator – but we aren't looking at the total circumference measure, we are looking at it in terms of the central angle; therefore, the denominator will always be either 360 (if working with degrees) or 2π (if working with radians)
    - What is the numerator?
      Answer: this is the measure of the central angle (either in degrees or radians) as well as the angle measure of the intercepted arc
  - Example: Hold the tennis ball back behind the head at approximately a 45-degree angle and demonstrate a throwing motion to approximately 45 degrees in front of the head – be sure students recognize the movement happening and how far the ball is traveling on the arc in terms of degrees and radians
    - Diagram this movement on the whiteboard with a graph, showing the starting point and the ending point to help visualize the 45 degrees + 45 degrees to make 90 degrees
    - Convert to radians: 90 degrees * $\frac{\pi}{180}$ = $\frac{\pi}{2}$ radians
  - Now, calculate the distance by measuring first the length of the throwing arm with a measuring tape → point out that this distance is the length of the radius and add this information to the diagram on the white board
    - If the throwing arm is approximately 30 in long, student will use this and

think back to what we're looking for: A fraction of the circumference
- C = 60π in, based on the throwing arm, but we aren't traveling the full circumference
- To find the distance traveled, we need to take the fraction of the circumference – how?
  - Construct a fraction using the angle and 360 degrees or 2π radians: $\frac{90}{360}$ for degrees or $\frac{\frac{\pi}{2}}{2\pi}$ for radians
  - Use multiplication of these fractions by the circumference to find the arc length: $(\frac{90}{360})60\pi$ or $\left(\frac{\frac{\pi}{2}}{2\pi}\right)60\pi$
  - Two different calculations, but both yield the same answer: $15\pi\ in$
    - Why are they the same? Because both fractions reduce to ¼, and ¼ of 60π is 15π.
- Why does this make the atlatl an effective tool?
  - Estimate how long it takes the arm to move through the 90 degree/$\frac{\pi}{2}$ radian throwing angle?
  - Use student estimation to calculate speed:
    - How do we calculate speed?
    - As a hint, can ask them how speed is measured in their car
      - Answer: $\frac{distance}{time}$
    - So for the arm throwing the tennis ball, the speed with which the ball would leave the hand is $\frac{15\pi\ in}{time}$ (for the sake of the lesson plan, imagine that the predicted time is .5 sec, so the calculation is $\frac{15\pi\ in}{.5\ sec}$ or $\frac{30\pi\ in}{sec}$ or ~94.2 in/sec
  - Now imagine throwing the ball with a *ChuckIt*! ball launcher. Have students measure the *ChuckIt*! ball launcher with a tape measure (~20 in). How long is the throwing radius for the same throw along the same angle with the *ChuckIt*! ball launcher used?
    - Answer: 30 + 20 = 50 in
    - How does this change the arc length and speed of the projectile?
      - Answer: Start with the circumference, which is is now $100\pi\ in$
      - To calculate the arc length, we will use both degrees and radians: $(\frac{90}{360})100\pi$ or $\left(\frac{\frac{\pi}{2}}{2\pi}\right)100\pi$ = 25π in
      - Ask: To cover 25π in in the same amount of time (.5 sec), will the ball travel faster or slower?
        - Answer: Faster

- Proof: $\frac{25\pi\ in}{.5\ sec}$ or $\frac{50\pi\ in}{sec}$ or ~157 in/sec
- This is much faster than just throwing the ball by hand
- Thus, by extending the length of the throwing arm (radius), a projectile is able to travel further

- Can you think of other devices that use this principle or something similar?
  - Possible suggestions: Catapults or trebuchets

- (~20 minutes) Now, let's generalize an equation for arc length
  - What did we use?
    - Circumference
    - Central angle
    - The radian or angle measure for a full rotation around a circle
  - General equation for arc length: A.L. = $(\frac{central\ angle}{360})(2\pi r)$ or A.L. = $(\frac{central\ angle}{2\pi})(2\pi r)$
  - How can we use this to find the area of a sector?
    - First question – What is a sector?



    - A *sector* is a portion of the circle that resembles a pizza slice or wedge – it is not the entire *area*, but is a part of it
    - Since it is part of the area, we can set up sector area similar to what we did with arc length, but instead of using circumference, we use the area formula:
      - Sector Area = $(\frac{central\ angle}{360})(\pi r^2)$ or $(\frac{central\ angle}{2\pi})(\pi r^2)$
  - (~10 minutes) Practice with questions 1-4 on the Arcs, Arc Length & Sector Area Assignment as a class. Have students work in groups of 2-3 to answer these four questions; come back together as a class to review answers and answer any last questions or clarify any confusion
  - (~20 minutes) Allow the remainder of class to be time for independent work on the assignment below

- *Assignment*: Questions 5-20 on the Arcs, Arc Length & Sector Area Assignment page (#17-20 will review a previous lesson) + the following question: At what length do you believe an atlatl would have become unwieldy to use? Provide a written explanation supported by mathematical calculations and be prepared to explain your answer tomorrow in class.

Materials:

- Class set of Chromebooks/laptops/iPads/etc
- A *ChuckIt*! ball launcher or similar instrument to demonstrate why the atlatl was so effective in increasing distance and speed of a projectile
- Measuring tape
- Desmos Vocabulary Matching Activity
- Arcs, Arc Length & Sector Area Assignment

This lesson plan was created by Stephanie Morgan of Pisgah High School as part of the 2021-2022 UNC World View Global Fellows Program. For more information about the program, please visit http://worldview.unc.edu/